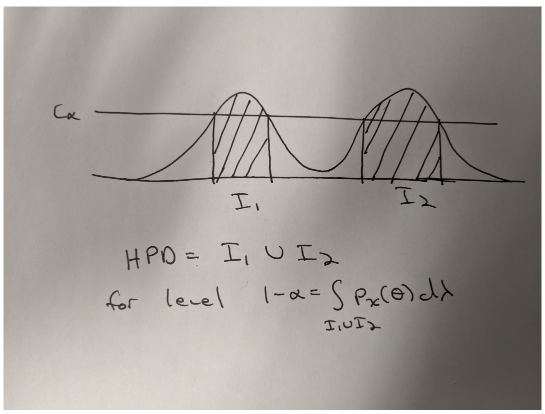
Rice STAT 533/ GS01 1283 Homework 4 Solutions
March 3, 2020
Request: Please email the instructor if you find any mistakes in this document.

Extra Question:

a. Let C be the HPD region and suppose $\theta_1, \theta_2 \in C$ and $\theta_1 < \theta_2$. Consider any θ such that $\theta_1 < \theta < \theta_2$. Since $p_x(\theta)$ is a density, $\lim_{\theta \to \infty} p_x(\theta) = 0$ and $\lim_{\theta \to -\infty} p_x(\theta) = 0$. If $p_x(\theta) < \min(p_x(\theta_1), p_x(\theta_2))$ then by the extreme value theorem there are modes in $(-\infty, \theta)$ and (θ, ∞). Since f is unimodal $p_x(\theta) \geq \min(p_x(\theta_1), p_x(\theta_2))$. Thus θ is in the HPD region. We have proved that the HPD region is convex. Convex sets in $\mathbb{R}$ are intervals.

b. See plot below.



c. By a) the HPD region is an interval. By symmetry $f(x_0 - a) = f(x_0 + a)$. Thus $x_0 - a \in C(X) \implies x_0 + a \in C(X)$ for any a. Therefore the interval is centered at x_0 and is of the form $[x_0 - a, x_0 + a]$. By symmetry $\int_{-\infty}^{x_0 - a} f = \int_{x_0 + a}^{\infty} f = \alpha/2$. Thus the HPD and percentile intervals will match.