Systematic Errors

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Systematic errors in astronomy is important as it determines how accurate one can achieve when making single or multiple observations of an event.

We will consider two types of systematic errors.

1 Multiplicative errors

This occurs from normalization of a measurement. For example, in errors of flat fielding, or unknown errors of flux levels of comparison stars.

Suppose data y_i are measurements of fluxes of an object (such as the light curve of a star), with statistical errors of σ_i. All the data points suffer from an unknown form of systematic error such that the flux levels of each measurement is accurate to ϵ. The flux need to be modified by an additional factor $\alpha = 1 \pm \epsilon$

$$f_i = \alpha f(x_i, \mathbf{a}).$$

The likelihood function is then

$$L = P(\mathbf{a}|y_i, \sigma_i, \alpha) \cdot P(\alpha)$$

which is

$$L = \Pi_i \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left(-\frac{y_i - \alpha f(x_i, \mathbf{a})}{2\sigma_i^2}\right) \cdot \frac{1}{\sqrt{2\pi}\epsilon} \exp\left(-\frac{(\alpha-1)^2}{\epsilon^2}\right)$$

From this,

$$\chi^2 = -\ln L = \Sigma_i \frac{y_i - \alpha f(x_i, \mathbf{a})}{2\sigma_i^2} + \frac{(\alpha-1)^2}{\epsilon^2}$$

If data are from more than one experiments, different α may be needed for each experiment.

The χ^2 minimization process will have to fit α as a nuisance parameter and marginalize over it.

2 Additive Errors

2.1 Model fits

Sometime the systematic errors are additive. Such as errors introduced from bias or sky subtraction. In this case the models will be different from multiplicative errors.

The model to the data can be written as

$$f_i \;=\; f(x_i, \mathbf{a}) + \beta,$$

which leads to

$$\chi^2 \;=\; \Sigma_i \frac{(y_i - (f(x_i, \mathbf{a}) + \beta))^2}{\sigma_i^2} + \frac{\beta^2}{\epsilon^2}$$

The parameter β can be taken as a nuisance parameter and it can be marginalized over to determine the parameter $\mathbf{a}$.

2.2 Covariance Matrix

Sometime astronomers sets an arbitrary error floor in the data, just because our knowledge is insufficient to determine errors to infinite precision, so that the noise cannot always be beaten down by a large number of repeating measurements.

If two measurements y_1 and y_2 have a common systematic error ϵ, and also individual random errors σ_1 and σ_2, the values of y_1 and y_2 can be considered as having two parts, y_1^R and y_2^R are the random component and y_1^S and y_2^S are the systematic component. The systematic component are correlated errors. The Covariance of y_1 and y_2, for example, is given by

$$Cov(y_1, y_2) \;= <(y_1^R + y_1^S)(y_2^R + y_2^R)>= Cov(y_1^R, y_2^R) + S^2$$

In general, the Covariance matrix

$$Cov(\mathbf{y}, \mathbf{y}) = Cov^R(\mathbf{y}, \mathbf{y}) + Cov^S \mathbf{s}\mathbf{s}^\mathbf{T}$$

Correspondingly,

$$\chi^2(\mathbf{a}) \;=\; \mathbf{\Delta}^T Cov(\mathbf{y}, \mathbf{y})^{-1} \mathbf{\Delta},$$

with

$$\mathbf{\Delta} = \mathbf{y} - \mathbf{f}(\mathbf{x}, \mathbf{a})$$