Homework 1
Due Date: September 17

1. Suppose brightness measurements $m_1, \ldots, m_n$ are made on a standard star with unknown mean magnitude μ. Every measurement is made with known uncertainty σ (same for each observation). The model is

$$m_i = \mu + \epsilon_i.$$

Typically in astronomy the measurement error is assumed to be normal, i.e. ϵ_i is normal with mean 0 and standard deviation σ. Here we suppose that the ϵ_i are independent and follow a mean 0 double exponential distribution

$$f(\epsilon) = \frac{1}{2\lambda} e^{-|\epsilon|/\lambda}. \tag{1}$$

This distribution has mean 0 and standard deviation $\lambda\sqrt{2}$. You can generate random numbers from the double exponential distribution using the `rdoublex` function in the R package `smoothmest`.

   (a) To familiarize yourself with the double exponential distribution, draw 1000 random samples from a mean 0, standard deviation 1 double exponential and 1000 random samples from a standard normal. Construct and plot density estimates for both samples. How do they compare? What sort of errors are more / less likely under the different distributions?

   (b) Verify analytically that $f(\epsilon)$ in Equation (1) has mean 0 and standard deviation $\lambda\sqrt{2}$.

   (c) Show that the maximum likelihood estimator for μ is the median of $m_1, \ldots, m_n$. (HINT: median $= \underset{\mu}{\text{argmin}} \sum |m_i - \mu|$.)

   (d) Simulation study: Let $\mu = 18$ and $\sigma = 0.3$. Simulate $n = 100$ observations from the model and compute the mean and median of the observations. Repeat this 1000 times. Construct densities of your 1000 estimates (separate density for mean and median). Which estimator is better?

   (e) Redo simulation using normal errors with $\mu = 18$ and $\sigma = 0.3$. Which estimator is better now?

2. Positions, Distances, and Angles. A recent supernova 2015N was found to at R.A. = 21:43:16.89, Decl. = +43:34:47.6 in the galaxy UGC 11797. The coordinates of the center of UGC 11797 is R.A. = 21h:43:35.0, Decl = 43:34:35.0. A hypothetical star is at position R.A. = 21:43:34.5, Decl. = +43:34:35.3. The errors of the coordinates of the supernova and the star are all 0.2 arcsec, the errors of the coordinates of the center of the galaxy are 0.5 arcsec.

   (a) Calculate the angular distance between the supernova and the galaxy, and discuss the error of the distance

(b) Calculate the angular distance between the star and the galaxy, and discuss the error of the distance

(c) The supernova, star, and the center of the galaxy form a triangle. Estimate the three angles of the triangle and discuss the errors associated with the three angles. How do the three angles correlate if the requirement of their sum being 180 degrees is enforced?