Causal Inference with Hidden Confounders

Instrumental Variables and the Generalized Causal Dantzig

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Outline

Hidden Confounding, Reverse Causality and Instrumental Variables

Generalized Causal Dantzig

Simulation

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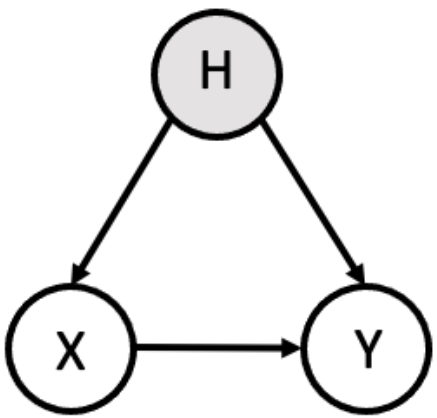
Hidden Confounders and Reverse Causality

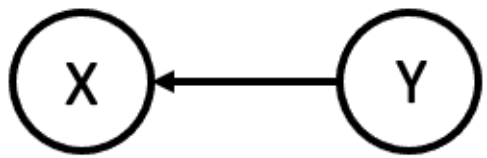
Goal: Estimate causal effect of X on Y.

Problem: Regression may produce inconsistent estimates.

Reason 1: Hidden confounding



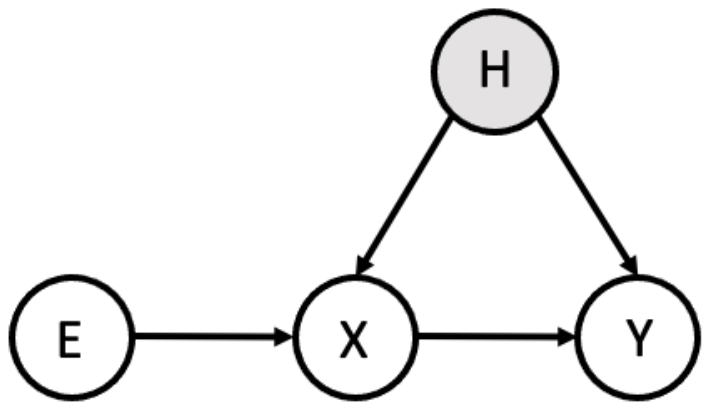
Reason 2: Reverse causality



Instrumental Variables (IV)

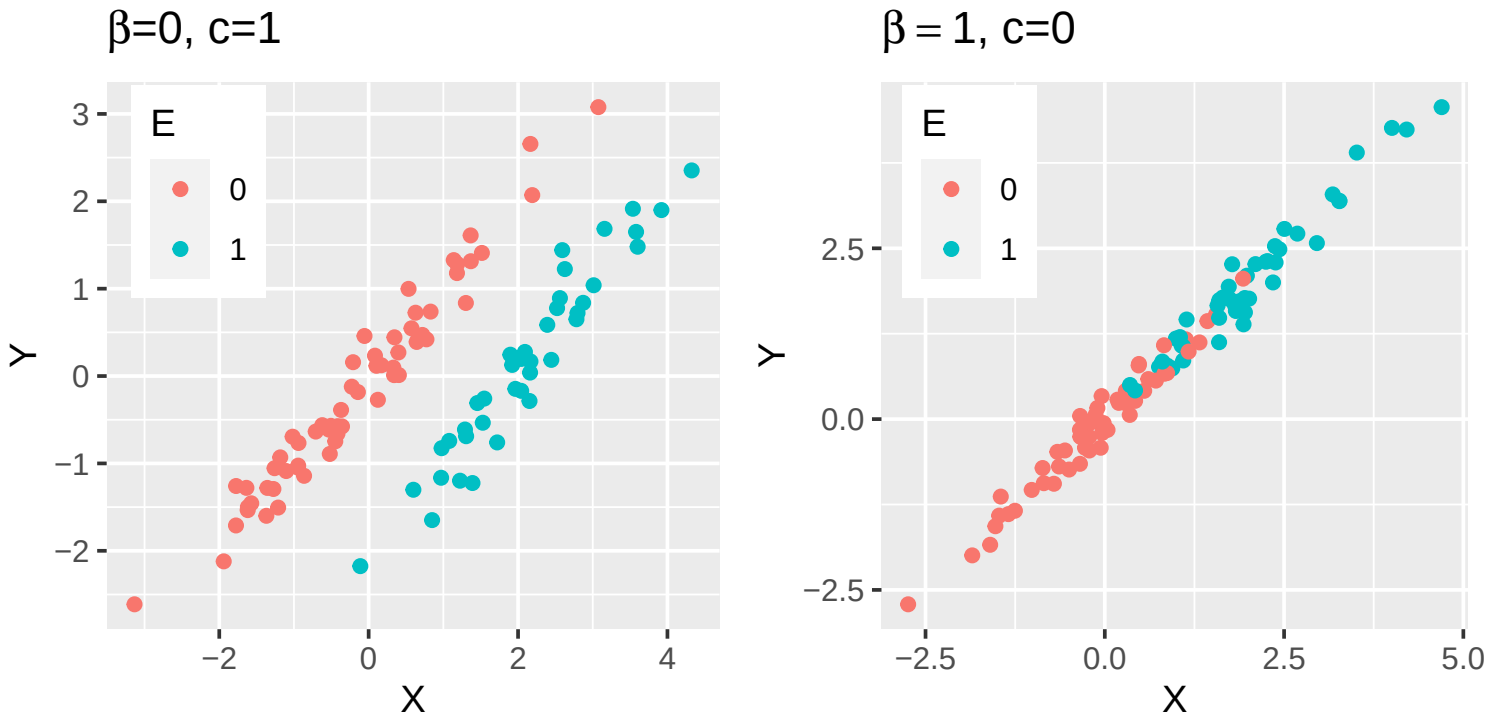
E is an instrument if:

1. correlated with exposure X
2. does not have a direct effect on Y



How Do IVs Identify Causal Effects?

$$E \sim Bernoulli(1/2)$$
$$X \leftarrow 2E + H + \epsilon_X$$
$$Y \leftarrow \beta X + cH + \epsilon_Y$$



IV Estimator via Generalized Method of Moments

- **Main Idea:**

$$E \perp \underbrace{Y - X^T \overbrace{\beta_0}^{\text{true } \beta}}_{=h+\epsilon_Y}$$

- Moment conditions:

$$m_{IV}(\beta) \equiv \mathbb{E}[E(Y - X^T\beta)] \underbrace{=}_{\text{iff } \beta=\beta_0} 0$$

$$\widehat{m}_{IV}(\beta) \equiv \frac{1}{n}\sum_{i=1}^{n} E_i(Y_i - X_i^T\beta) \underbrace{\approx}_{\text{when } \beta=\beta_0} 0$$

- Estimate β with:

$$\widehat{\beta}_{IV}(\widehat{W}) = \underset{\beta}{\text{argmin}} \; ||\widehat{m}(\beta)||_{\widehat{W}} = \underset{\beta}{\text{argmin}} \; \widehat{m}(\beta)^T \widehat{W} \widehat{m}(\beta)$$

- Two Stage Least Squares (TSLS) uses $\widehat{W}$ which is optimal under some conditions.

IV References: IVs date back to Wright [1928]. See Mátyás [1999] for GMM estimation in IV models.

Overview of Remainder of Talk

Background on Causal Dantzig (CD):

- Proposed in Rothenhäusler et al. [2019] AOS
- Consistent under hidden confounding

Contributions:

- New Estimator: Generalized Causal Dantzig (GCD)
- GCD and IV Comparisons

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Environments and CD Response Model

- (X, Y) data may be collected in different **environments**
  - Example: Perturbation Biology
    - Environment 0: No stimulation
    - Environment 1: Stimulation with reagents
    - Data: Protein expression (X) and phenotype (Y) for many cells in each environment
- Goal: Use environment to infer $X \to Y$ causal effects
- Several environment based estimators have been proposed:
  - Invariant Causal Prediction (ICP) Peters et al. [2016], Sequential ICP Pfister et al. [2019], Nonlinear ICP Heinze-Deml et al. [2018]
  - **Causal Dantzig**
- Main Assumptions of Causal Dantzig:

$$\underbrace{Y^e}_{\text{env. e resp.}} = \overbrace{X^{e^T}}^{\text{env. e exp.}} \beta + \delta_Y$$

Causal Dantzig (CD) Estimator

Let:

- $\mathbf{X^j} \in \mathbb{R}^{\mathbf{n_j} \times \mathbf{p}}$ be the environment j design matrix
- $\mathbf{Y^j} \in \mathbb{R}^{\mathbf{n_j}}$ be the environment j response

With 2 environments the CD is:

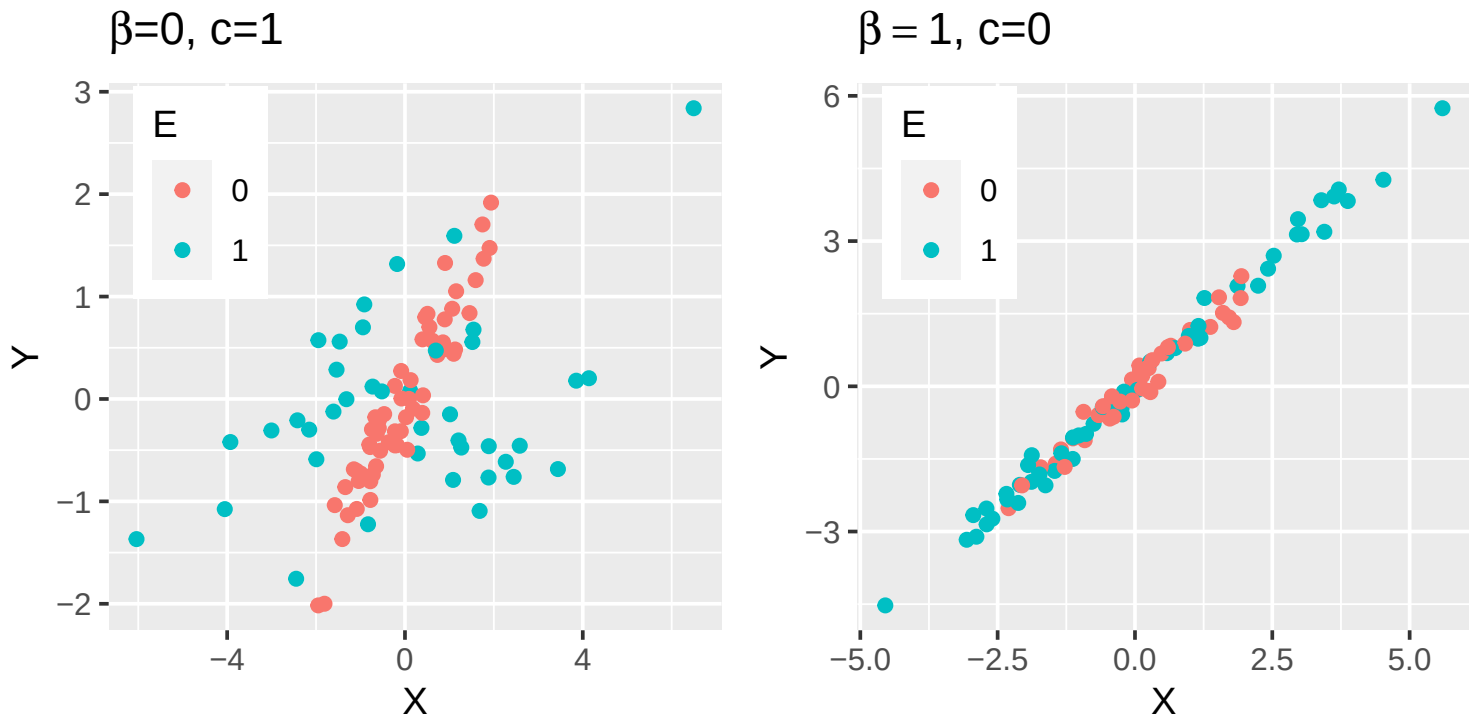
$$\widehat{\beta}_{CD} = \left(\overbrace{\frac{1}{n_1}\mathbf{X}^{1^T}\mathbf{X}^1 - \frac{1}{n_0}\mathbf{X}^{0^T}\mathbf{X}^0}^{\Delta \text{ 2nd moment of exposures}} \right)^{-1} \left(\underbrace{\frac{1}{n_1}\mathbf{X}^{1^T}\mathbf{Y}^1 - \frac{1}{n_0}\mathbf{X}^{0^T}\mathbf{Y}^0}_{\Delta \text{ exposure} \times \text{ response}} \right)$$

Notes:

- Proposed in Rothenhäusler et al. [2019]
- Motivated by notion of inner product invariance

When Can CD Identify Causal Effects?

$$E \sim Bernoulli(1/2)$$
$$X \leftarrow H + (2E + .2)\epsilon_X$$
$$Y \leftarrow \beta X + cH + \epsilon_Y$$



Note: E does not shift mean of X, so IV/TSLS inconsistent.

Notes on CD

Strengths:

- CD is consistent for some models where TSLS fails (e.g. variance shifts)
- Only need two environments to identify causal effects (under assumptions on how environment effects $X \in \mathbb{R}^p$)

Weaknesses:

- Environments must be discrete.
- Rothenhäusler et al. [2019] has no theory or optimality guarantees for estimators with more than 2 environments.

Generalized Causal Dantzig (GCD) Estimator

New Estimator: The GCD is a GMM with moment conditions:

$$m_{GCD}(\beta) \equiv \mathbb{E}[vec(EX^T)(Y - X^T\beta)].$$

- $E \in \mathbb{R}^q$ is set of instruments.
- $vec(EX^T) \in \mathbb{R}^{qp}$ column stacks matrix EX^T
- $\widehat{\beta}_{GCD}(\widehat{W}) = \underset{\beta}{\text{argmin}} \; ||\widehat{m}_{GCD}(\beta)||_{\widehat{W}}$

GCD Estimator

- The GCD exactly matches CD in 2 environment case.
  - Idea: Encode (categorical) environment $\mathcal{E} \in \{0, 1, 2, \ldots\}$ as instrument $E \in \mathbb{R}^{\#\mathcal{E}-1}$.
- GCD can be used with continuous environments, CD cannot.
- GMM theory can be used to optimally weight environments (2 step estimators).
- Conceptual Result: Environments and instruments are very closely related.

$$m_{IV}(\beta) = \mathbb{E}[E(Y - X^T\beta)]$$
$$m_{GCD}(\beta) = \mathbb{E}[vec(EX^T)(Y - X^T\beta)]$$

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Simulation Parameters

Exposure and response models:

$$X \leftarrow 9h + (10E + 1)\epsilon_X$$
$$Y \leftarrow 3h + X + \epsilon_Y$$

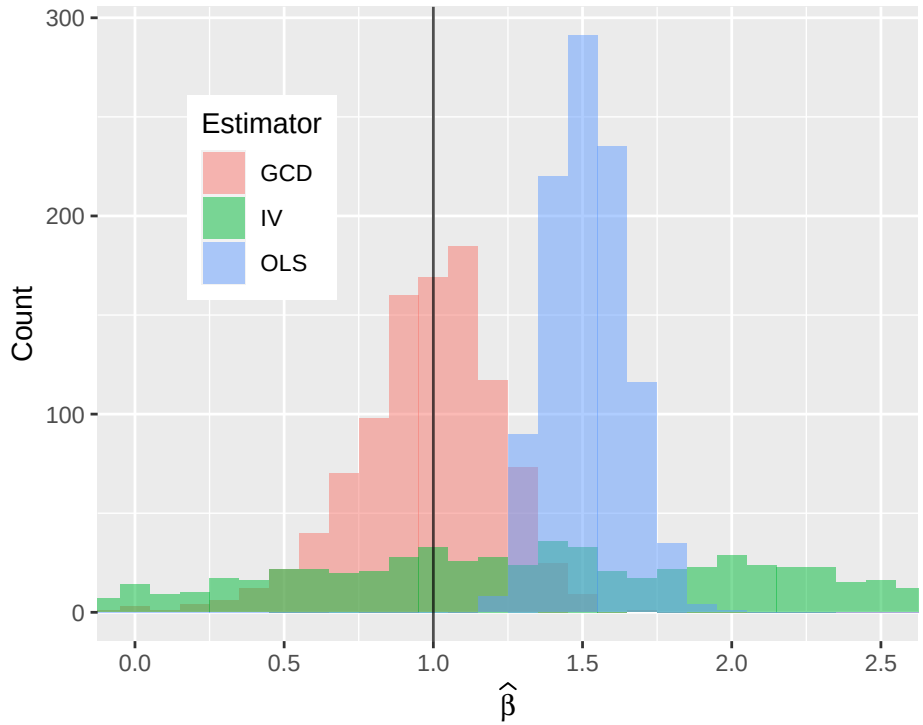
Exogenous variables (all independent) are:

$$E \sim Unif[0, 1]$$
$$h, \epsilon_X, \epsilon_Y \sim N(0, 1)$$

Sample size $n = 100$.

Fit 3 estimators $\widehat{\beta}_{GCD}$, $\widehat{\beta}_{IV}$, $\widehat{\beta}_{OLS}$.

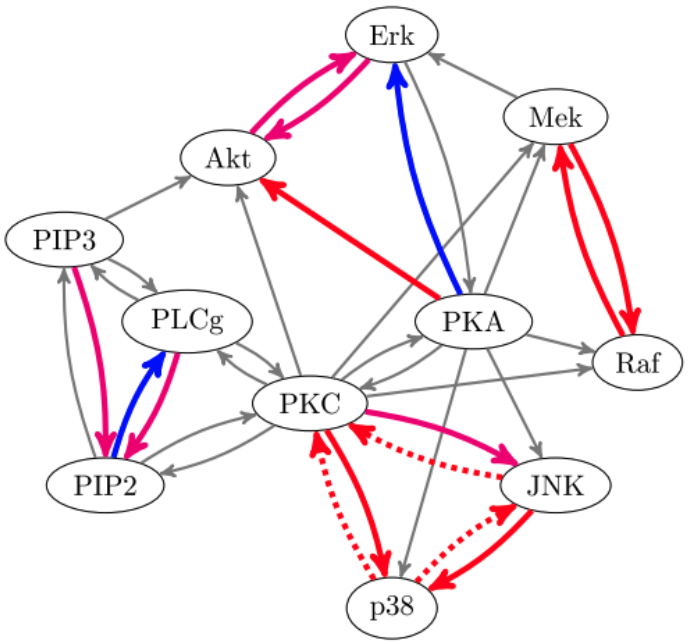
Empirical Sampling Distributions



- OLS is inconsistent due to hidden confounding.
- IV inconsistent because E does not shift mean of X.
- GCD performs well.
- CD not applicable because instrument is continuous.

Ongoing Work

- Currently working on applications in causal discovery with Flow Cytometry Data



- Manuscript available on arxiv soon

Source: Meinshausen et al. [2016]

Thank you. Questions?

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